Discrete Choquet integral based method for criteria synergy determination

Rubén Bernal – Marcelo Karanik
National Technological University
Chaco, Argentine
rbernal@frre.utn.edu.ar, marcelo@frre.utn.edu.ar

José Ignacio Peláez Sánchez
Department of Languages and Computer Sciences
University of Malaga
Málaga, Spain
jignacio@lcc.uma.es

Estela del Rocio Yanez Benavides
University of Guayaquil
Ecuador
estela.yanezb@ug.edu.ec

***Abstract*—In multi-criteria decision-making processes, a set of criteria that can act independently or have some kind of relation between them are considered. When this latter appears, the process cannot be a simple problem due to complexity to model such synergy relations. To deal with these issues, a Choquet integral based method has been developed to determine the most appropriate ranking of alternatives. Therefore, a fuzzy measure that models considerations of an expert in the problem domain must be identified. The proposed model is a novel and efficient algorithm for determining a non-additive fuzzy measure guided by linguistic attributes. Finally, an illustrative example and a comparison with other aggregation operators is presented.**

Keywords—Fuzzy Measure; Choquet Integral; Multiple Criteria Decision Analysis.

I. INTRODUCTION

Decision-making (DM) is a very important activity related to choosing the most adequate option among the ones available. Multi-criteria decision-making (MCDM) process uses several techniques to select the most appropriate alternative in order to solve complex problems. This alternative is selected from the alternatives set evaluated with respect to several criteria or attributes. It can become extremely complex, especially when uncertainty environments, such as risk or imprecise data situations, are involved.

In many situations, adequateness ranking of alternatives can be wrong if dependence between criteria is not considered.

Clearly, this type of decision process involves evaluating several alternatives based on multiple criteria with dependence relations between them. This dependence generates groups that have more or less weight (importance) than the sum of individual ones (positive or negative interaction or synergy) [1]. To model this phenomenon, the weight vector of the weighted arithmetic mean can be replaced by a function of non-additive sets (on evaluation criteria set) called fuzzy measure. Thereby, it is possible to define a weight for each criterion and each subset of criteria. Many examples in the literature deal with this trouble; e.g., consumers preference analysis [2], estimations in project management [3], hotel selection preferences of Hong Kong inbound travelers [4], valuation of residential properties [5].

In problems with independent criteria, i.e. when there is no relation between them, the importance of the groups of criteria is additive. However, for many practical applications, the individual criterion has some interaction with other/s and traditional method/s (such as weighted arithmetic mean or OWA operators) are not adequate to deal with this situation [6].

There are different models to determine fuzzy measure with a lot of techniques such as genetic algorithms, gradient descent algorithms, neural networks, particle swarm algorithm and even, by requiring some type of information from experts. Grabisch introduced a gradient descendent algorithm with constraints for identifying fuzzy measure applied to pattern recognition [7]. In [1] two approaches were presented (based on minimization of squared error and constraint satisfaction). Wang proposed a speculative algorithm to extract a fuzzy measure from sample data [8]. Other researchers determine fuzzy measures from sample data with genetic algorithms [9] and use random generation algorithms as a special case of generating points in an order polytope [10]. In an effort to generate methods or techniques to learn distinct fuzzy measures, a particle swarm algorithm to determine a type of general fuzzy measures from data is proposed [11].

All the methods mentioned above can determine a fuzzy measure based on sample data. Complementary to this class of methods, other techniques that require some information from different experts are proposed. For example, in [12], the decision maker must identify the weights and interaction degrees among criteria using a labeled diamond.

The main purpose of this paper is to propose a method (based on discrete Choquet integral) to establish the importance of individual criterion and the possible groups, when dependence exists. A computing with words method is used to obtain that fuzzy measure based on knowledge of different experts. Linguistic labels [13] [14] [15] and preference relations to establish the importance (weight) of each criterion and groups of them are used. In this proposed method, the experts are not required to boast a high level of accuracy to prioritize criteria, but a more flexible way to express their assessment is allowed. The discrete Choquet integral regarding non-additive (or fuzzy) measures, is an aggregation operator suitable for modeling the phenomenon above exposed [16].

The structure of this paper is as follows: In Section II some basic definitions such as fuzzy measure, Choquet integral and general considerations are presented. In Section III, a new model for constructing a fuzzy measure is described, including an illustrative example. In Section IV the results of the model, WAM and OWA operators are compared. Finally, concluding remarks are exposed in Sections V.

II. Preliminary Definitions

A. Aggregation functions

Aggregation functions are special real functions with inputs from a subdomain $\mathbb{I}$ of the extended real line. The basic feature of all aggregation functions is their non-decreasing, monotonicity and boundary conditions. Increasing of input values cannot decrease the output values and they are aggregated in the same scale of input values, respectively.

Formally [17], an *aggregation function* is a function $A^{(n)}: \mathbb{I}^n \to \mathbb{I}$ that:

i. is non-decreasing (on each variable);
ii. fulfills the boundary conditions;

$$\inf_{x \in \mathbb{I}^n} A^{(n)}(\boldsymbol{x}) = \inf \mathbb{I} \quad \text{and} \quad \sup_{x \in \mathbb{I}^n} A^{(n)}(\boldsymbol{x}) = \sup \mathbb{I} \qquad (1)$$

$$\text{If } \mathbb{I} = [a,b],\ A^{(n)}(\mathbf{a}) = a \text{ and } A^{(n)}(\mathbf{b}) = b \qquad (2)$$

where $\mathbf{a} = (a, \ldots, a)$ and $\mathbf{b} = (b, \ldots, b)$

iii. for all $x \in \mathbb{I}$

$$A^{(1)}(x) = x \qquad (3)$$

The integer n represents the cardinality of the aggregation function, i.e. the number of its variables. Examples of basic aggregation functions are the arithmetic mean, geometric mean, minimum function, maximum function, OWA [18], MA-OWA [19], etc.

The integral concept (defined with respect to a measure) is another generalization of aggregation function. Fuzzy measures and fuzzy integrals have been applied to many fields, such as MCDM because they can be considered as an aggregation operator that currently constitutes a key research topic.

B. Fuzzy measure

Let N a classical set and let $\mathcal{A}$ a σ-algebra of subsets of N; a fuzzy measure [1] on N is a monotonic set function $\mu: \mathcal{A} \to [0,1]$ satisfying:

$$\mu(\varnothing) = 0; \qquad \mu(N) = 1 \qquad (4)$$

$$\mu(A) \leq \mu(B) \ \ \forall A, B \in \mathcal{A} \text{ such that } A \subseteq B \qquad (5)$$

A fuzzy measure assigns weight of importance to all possible subsets of criteria from a given criteria set. Thus, in addition to the assignment of weights for individuals, weights assigned to any combination of criteria are also defined. A fuzzy measure μ is *additive* if $\mu(A \cup B) = \mu(A) + \mu(B)$ whenever $A \cap B = \varnothing$.

Generally, in problems of MCDM, criteria used to evaluate a set of alternatives have some kind of interaction such as correlation, substitutiveness/complementarity and preferential dependence. These relations are usually very complex and identifying them is difficult [20]. Often, they are modeled with sub-additive or super-additive fuzzy measures. A positive (direct) correlation between criteria i and j must be modeled by $\mu(ij) < \mu(i) + \mu(j)$ which expresses negative interaction or negative synergy; that is, "the marginal contribution of j to every combination of criteria that contains i is strictly lesser than the marginal contribution of j to the same combination when i is excluded" [20].

A negative (inverse) correlation between criteria i and j must be modeled by $\mu(ij) > \mu(i) + \mu(j)$ which expresses positive interaction or positive synergy; that is, "high partial scores along i usually imply low partial scores along j and vice versa. Simultaneous satisfaction of both criteria is rather uncommon; therefore, the alternatives that present such a satisfaction profile should be favored" [20].

C. λ-Fuzzy measure

A λ-fuzzy measure [21] is a fuzzy measure μ that for all $A, B \in N, A \cap B = \varnothing$ satisfies:

$$\mu(A \cup B) = \mu(A) + \mu(B) + \lambda\mu(A)\mu(B) \qquad (6)$$

where $\lambda \in (-1, \infty)$.

Varying λ conveniently according to the information provided by the expert, it is possible to determine a fuzzy measure to model such preferences. It measures the criteria interaction intensity and varies (proportionally) with the number of linguistic labels that the expert can distinguish. When the value of λ is selected, boundary and monotonicity conditions must be taken into account. Specifically:

i. Negative interaction:

$$\text{If } \lambda \in (-1, 0),\ \mu(A \cup B) < \mu(A) + \mu(B) \qquad (7)$$

ii. Positive interaction:

$$\text{If } \lambda \in (0, \infty),\ \mu(A \cup B) > \mu(A) + \mu(B) \qquad (8)$$

iii. No interaction:

$$\text{If } \lambda = 0,\ \mu(A \cup B) = \mu(A) + \mu(B) \qquad (9)$$

where $A, B \in N$.

In (9) the fuzzy measure behaves like the weight arithmetic mean, while in (7) and (8) negative and positive synergy is, respectively, observed.

Consider two criteria c_1 and c_2, used to evaluate and to rank several alternatives. If one of these alternatives satisfies only one single criterion, the decision-maker may consider that it is equally worthless as if it satisfies none. In this case c_1 and c_2 act conjunctively, so that both can be satisfied (positive interaction). This implies that the importance of a single criterion for the decision is almost zero while that of the pair is highly important. The criteria can be said to be complementary ($\lambda > 0$).

On the contrary, if one alternative satisfies only one single criterion, e.g., c_1, the decision-maker may consider that it is equally good as another that satisfies both. In this case, c_1 and c_2 act disjunctively, and it is sufficient that one of them is satisfied (negative interaction). The union of criteria has no additional benefit, and the importance of the pair is almost the same as the importance of the single criteria. In such case, they are said to be redundant ($\lambda < 0$).

If the decision-maker thinks that the importance of the pair is approximately the sum of the individual importance of criteria; they act independently and there is no interaction between them [16].

Fig. 1 shows the way to determine the value of $\mu(A \cup B)$ considering the variation of λ and the level of satisfaction (score) for each criterion.

As mentioned above, to maintain the monotonicity of the fuzzy measure, the value of λ must be in the range $[\lambda_{min}, \lambda_{max}]$, where

$$\lambda_{min} = \frac{max[\mu(A), \mu(B)] - \left(\mu(A) + \mu(B)\right)}{\mu(A)\mu(B)} \tag{10}$$
$$A, B \in \mathcal{P}(N)$$

and[1]

$$\lambda_{max} = abs(\lambda_{min}) \tag{11}$$

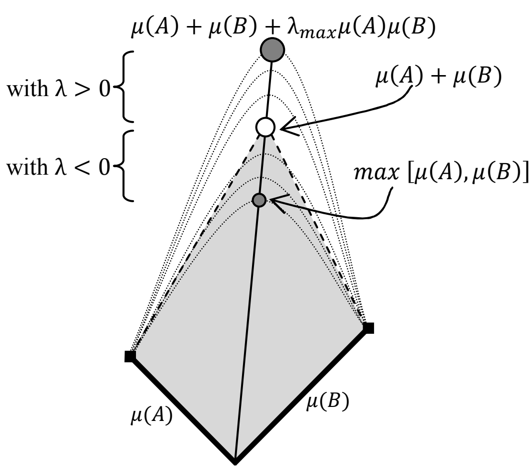

Fig. 1. $\mu(A \cup B)$ Depending on the variation of λ

[1] To keep equidistance to 0.

D. Choquet integral

The Choquet integral was introduced in 1954 by Gustave Choquet [22]. It can be considered as an aggregation operator, which generalizes the weighted arithmetic mean and is able to represent dependence between criteria in many situations.

Let μ be a fuzzy measure on $N = \{c_1, \dots, c_n\}$ and $\mathbf{x} = (x_1, x_2, \dots, x_n) \in [0, \infty)^n$, the (discrete) Choquet integral of $\mathbf{x}$ with respect of μ is defined by:

$$\mathbb{C}_\mu(\mathbf{x}) = \sum_{i=1}^{n} x_{(i)}\left[\mu\left(A_{(i)}\right) - \mu\left(A_{(i+1)}\right)\right] \tag{12}$$

where $x_{(\cdot)}$ indicates a permutation such that $x_{(1)} \le x_{(2)} \le \dots \le x_{(n)}$; $A_{(i)} = \{i, \dots, n\}$; $A_{(n+1)} = \emptyset$; $\mu\left(A_{(n+1)}\right) = 0$.

The Choquet integral has very good properties of aggregation [20]. For instance, *increasing monotonicity* (non-negative response to any increase of the arguments); *idempotence* (comprised between min and max); *stability* with respect to the same interval scales and coincides with the weighted arithmetic mean when the fuzzy measure is additive (see e.g. [20], [17]).

E. Linguistic Labels

Information can be expressed in a quantitative or qualitative way. Normally, it is assumed that dealing with quantities is easier than dealing with qualities. Sometimes this is not true, specifically when perceptions over quantities are required. Humans have an intrinsic capability to perform a wide variety of physical and mental tasks without any measurements and any computations. Indeed, this capability allows decisions under uncertainty. In order to choose the best alternative available, decision-makers manipulate different kinds of perceptions. This process involves human recognition, decision and execution processes [23].

In order to express perceptions, humans use words representing estimates. These words play the role of labels of perceptions. More generally, perceptions are expressed using several linguistic computational models [24].

Linguistic labels are mathematical functions used for assigning values to concepts. These values belong to a numerical scale previously defined. The number of elements in the term set establishes the granularity of the uncertainty (see [25]). The decision-maker uses these labels without knowledge of what values they represent. The use of several linguistic labels sets gives more flexibility, i.e., one expert may choose values from 1 to 5 as his evaluation [{none, low, medium, high, perfect}], while another expert may use 7 labels [{none, very low, low, medium, high, very high, perfect}] to make a more detailed opinion. Clearly, using less linguistic labels simplifies the information representation and by using more linguistic labels, the correctness of the decision is improved, thus increasing the computational complexity.

In this work, linguistic labels are used for two purposes: (1) to assign the importance level for each criterion and (2) to assign synergy degree of criteria groups. For both, the expertise *level* defines the label granularity.

III. PROPOSED MODEL

An interesting challenge arising in the practical use of fuzzy measures is to determine a fuzzy measure model that deals with concrete situations. A problem involving n criteria, requires $2^n - 2$ coefficients[2] in order to define a fuzzy measure μ. Clearly, to provide this amount of information is very hard and usually the meaning of the values $\mu(A)$ is not sufficiently clear for the experts.

In the proposed method it is not necessary for the experts to provide the synergy degree of all combination of criteria. Only the obvious relationships are necessary; the rest is considered additive. With these considerations, the proposed model computes an adequate fuzzy measure and the expert can regulate the interaction degree by specifying different linguistic labels.

Let´s consider $A = \{a_1, \dots, a_k\}$ a set of alternatives to be evaluated with respect to a set of n criteria $N = \{c_1, \dots, c_n\}$. Each alternative $a_i \in A$, has a profile $x^a = (x_1^a, x_2^a, \dots, x_n^a) \in \mathbb{R}^n$, where x_i^a is a partial valuation of a w.r.t. the criterion c_i. From x^a it is possible to calculate an overall measure $M(x^a)$ for each alternative by an aggregation operator $M: \mathbb{R}^n \to \mathbb{R}$. Also consider A, B, C ... the subsets $\in \mathcal{P}(N)$ and $\mu(A), \mu(B)$, ... their weights.

To identify the weight of each subset of criteria, two steps are considered: (a) *Determine the importance of criteria:* each criterion will be evaluated individually to determine its relative importance with respect to the objective. Each $\mu(C_i)$, with $C_i \in N$ and $1 \le i \le n$ will be determined. (b) *Determine the weight of interacting criteria:* the experts will be asked for the sign and degree of interacting criteria in order to determine the weight of each coalition. Linguistic labels are used to establish the importance (weight) of each group of criteria. Each $\mu(C_i), C_i \in \mathcal{P}(N) - N$ will be determined, where $\mathcal{P}(N)$ is the power set of N.

It aims to build a fuzzy measure $\mu: \mathcal{P}(N) \to [0,1]$ such that:

$\mu(\emptyset) = 0$;
$\mu(N) = 1$;
$\mu(A) \le \mu(B)$ if $A \subseteq B, \forall A, B \in \mathcal{P}(N)$

Let´s also consider:

- a set of labels $V = \{v_1, \dots, v_j\}$; each v_i is used to provide the relative importance of each criteria w.r.t. the objective being evaluated;[3]
- a set of labels $W = \{w_1, \dots, w_j\}$; each w_i is used to provide the synergy degree between criteria including the null label;[4]
- $el \in [1..3]$ that characterizes each expert according to its expertise level;
- a pair (el, lq) indicating the number of different linguistic labels of V that the expert can distinguish; the values of lq will be set depending on the expertise level el.

  if $(el, lq) = (1, 3)$:
  $V = \{$*low, medium, high*$\}$ and
  $W = \{$*null, moderate, extreme*$\}$;
  if $(el, lq) = (2, 5)$:
  $V = \{$*very low, low ,medium, high, very high*$\}$ and
  $W = \{$*null, weak, moderate, strong, extreme*$\}$;
  if $(el, lq) = (3, 7)$:
  $V = \{$*lowest, very low, low, medium, high, very high, highest*$\}$ and
  $W = \{$*null, very weak, weak, moderate, strong, very strong, extreme*$\}$.

- a pair (e, lq) with the characterization profile of the expert e.

Algorithm

Determine a fuzzy measure μ.

1. Determine the number of labels lq that the expert e can distinguish.
2. Determine the weight of each criterion individually:
   a. Associate a label to each criterion on N.
   b. Compute the corresponding weight for each criterion.
3. For each $C \in \mathcal{P}(N)$ (if coalitions between criteria appear) do:

   Begin

   a. Determine the weight of coalition: for $C \in \mathcal{P}(N)$ with cardinality $1 < i \le n$:
   $\mu(C) = \mu(A) + \mu(B) + \lambda\mu(A)\mu(B)$
   where
   $\mu(A) = max\left(\mu(A_j)\right);\ A_j \subseteq C;\ |A_j| = i - 1$
   $\mu(B) = \mu(C - A)$
   $\lambda \in [\lambda_{min}, \lambda_{max}]$
   with
   $$\lambda_{min} = \frac{max[\mu(A), \mu(B)] - \left(\mu(A) + \mu(B)\right)}{\mu(A)\mu(B)};$$
   $\lambda_{max} = abs(\lambda_{min})$
   $A, B \in \mathcal{P}(N)$

   b. Establish a mapping between each linguistic label (of W) and $\mathbb{N}_0$, determine the synergy sign (positive or negative) and compute the λ value:
   for (−) synergy: $\lambda = -n\,\Delta_\lambda;\ \ 0 \le n \le (lq - 1)$
   for (+) synergy: $\lambda = n\,\Delta_\lambda;\ 0 \le n \le (lq - 1)$
   with: $\Delta_\lambda = \frac{\lambda_{max}}{lq-1}$

   End

[2] The coefficients of $\emptyset$ and N are determined by definition: $\mu(\emptyset) = 0$; $\mu(N) = 1$

[3] V does not include the Null label because a criterion with weight equal to 0 should not be taken into account for the decision process.

[4] W includes the Null label because there may exist criteria without interaction.

4. Normalize the values to $[0..1]$ to obtain μ:

$$\mu(A_i) = \frac{\mu(A_i)}{max\left(\mu(B)\right)}; \quad \forall A_i, B \in \mathcal{P}(N)$$

5. For each alternative $a_i \in A$, determine the profile:

$$x^a = (x_1^a, x_2^a, \dots, x_n^a) \in \mathbb{R}^n .$$

6. Aggregate results:
   a. For each alternative $a_i \in A$, compute the discrete CI $\mathbb{C}_\mu(a_i)$ w.r.t. fuzzy measure μ.

IV. ILLUSTRATIVE EXAMPLE

A. *Definition*

In this section, a problem of evaluation of people in their job performance according to four criteria is presented.

The evaluation criteria are defined accordingly: if the person is extrovert, interpersonal relationships with peers, technical skills, and readiness to transmit their knowledge.

c_1 (E) *Extroversion*: it refers to a person who is friendly and outgoing. Each candidate is rated on a scale of 1 to 10, evaluating characteristics such as friendliness, activities, social influence, positive emotions, etc.

c_2 (IR) *Interpersonal Relationships*: it is a close association between two or more people. Love, solidarity, regular business interactions or some other activity may influence in this type of relations.

c_3 (TS) *Technical Skills*: it is the level of knowledge and skill relating to a specific field. Each candidate is rated on a scale of 1 to 10 according to several theoretical evaluations.

c_4 (KT) *Knowledge Transfer*: it is the practical problem of transferring knowledge from one person to another; seeks to organize, create, capture or distribute knowledge to ensure its availability for future users.

Considerations and point of view of the expert are summarized in Sections B and C.

B. *Importance of criteria*

For each criterion mentioned in section A, the relative importance with respect to the objective to be tested was determined. An expert with $el = 2$ is considered, thus five linguistic labels are used. With $(el, lq) = (2,5)$:

$$V \to \mathbb{N} = \left\{ \begin{matrix} \textit{very low}\to 1, \textit{low}\to 2, \textit{medium}\to 3, \\ \textit{high}\to 4, \textit{very high}\to 5 \end{matrix} \right\}$$

Technical skills and how knowledge is transferred are the most important criteria with respect to personality. However, a candidate with a balanced profile between technical knowledge and readiness to transmit his knowledge will be taken into account. The following assignment of weights was made to the proposed criteria:

c_1: Extroversion (E)	*Low*	0.143
c_2: Interpersonal Relationships (IR)	*Medium*	0.214
c_3: Technical Skills (TS)	*Very High*	0.357
c_4: Knowledge Transfer (KT)	*High*	0.286

which allows to obtain the weight vector:

$w = (0.143, 0.214, 0.357, 0.286)$.

Three candidates, whose performance on each criterion is given in Table I, are considered.

TABLE I. PERFORMANCE OF THE DIFFERENT CANDIDATES

Candidate	E	IR	TS	KT
Cand1	6	7	8	2
Cand2	7	8	8	1
Cand3	7	8	5	4

C. *Weight of interacting criteria*

If a person is outgoing, they usually have good interpersonal relationships and vice versa. These two criteria have some degree of redundancy (overlap); therefore, the global evaluation will be overestimated (respectively underestimated) by extroverts (respectively introverts).

This phenomenon between Extroversion (c_1) and Interpersonal Relationship (c_2), can be easily overcome by using a fuzzy measure μ and the Choquet Integral $\mathbb{C}_\mu$ such that: $\mu(E, IR) < \mu(E) + \mu(IR)$ that expresses a negative synergy.

In the same way, due to the fact that most people have deeply ingrained the win/lose mentality from birth and think in terms of either/or in competitive situations, Technical Skills (c_3) and Knowledge Transfer (c_4) are negatively correlated. That is, high partial scores along c_3 usually imply low partial values along c_4 and vice versa. The simultaneous satisfaction of both criteria is rather uncommon (it involves mutual learning, mutual benefits and a win/win mentality); therefore, the alternatives that present such a satisfaction profile should be favored, i.e.: $\mu(TS, KT) > \mu(TS) + \mu(KT)$ that expresses a positive synergy.

Mapping of synergy scale is defined as follows:

$$W \to \mathbb{N}_0 = \left\{ \begin{matrix} \textit{null}\to 0, \textit{weak}\to 1, \textit{moderate}\to 2, \\ \textit{strong}\to 3, \textit{extreme}\to 4 \end{matrix} \right\}$$

The synergy degree assigned by the expert is "*strong*$(-)$" for the coalition $\{E, IR\}$ and "*extreme*$(+)$" for the coalition $\{TS, KT\}$.

With these considerations and by using the algorithm described above (steps 3 and 4), a fuzzy measure μ was automatically constructed (Table II).

D. *Results*

Using the Choquet Integral as aggregation operator (step 6 of algorithm) with respect to μ, global scores was obtained.

The analysis of the results is performed by comparing the results obtained by using the WAM, OWA and Choquet Integral operators. Table III summarizes the obtained results.

TABLE II. FUZZY MEASURE μ

Coalitions	Weights
{E}	0.120
{IR}	0.180
{TS}	0.300
{KT}	0.240
{E, IR}	0.240
{E, TS}	0.420
{E, KT}	0.360
{IR, TS}	0.480
{IR, KT}	0.420
{TS, KT}	0.700
{E, IR, TS}	0.600
{E, IR, KT}	0.540
{E, TS, KT}	0.820
{IR, TS, KT}	0.880
{E, IR, TS, KT}	1.000

TABLE III. RANKING OF ALTERNATIVES OBTAINED WITH WAM, OWA AND CHOQUET INTEGRAL

Candidate	E	IR	TS	KT
Cand1	6	7	8	2
Cand2	7	8	8	1
Cand3	7	8	5	4

$\mathbf{WAM_w}$	Rk	$\mathbf{OWA_w}$	Rk	$\mathbb{C}_\mu$	Rk
5.7857	2	6.3571	3	5.1800	2
5.8571	1	6.7857	1	5.0800	3
5.6429	3	6.4286	2	5.2000	1

Using the Weight Arithmetic Mean and OWA operator, the best candidate is candidate 2, because, on average, he/she has the best partial values. The optimistic OWA operator used in the example, improves results and allows the candidate 3 goes up in the ranking. However, results are not satisfactory because there is no operator that takes into account the interacting criteria.

On the contrary, by using the CI, candidates 1 and 2 lose their positions because they are the most unbalanced for the criteria c_3 and c_4 . Candidate 3 goes up two positions (comparing with WAM) and one position (comparing with OWA operator), who appears to be the best ranked when correlated criteria were considered (due to balanced scoring in all criteria). In addition, there was a decrease in the global score of all alternatives because criteria c_1 and c_2 have negative synergy. This avoids overestimates in high scores.

V. CONCLUSIONS

A new model to identify non-additive fuzzy measures based on linguistic labels was presented in this article. The proposed model allows the decision-maker to be focused on individual criteria and their relations, reducing the judgment process notably. Decision-maker categorizes criteria groups and the interaction type between them using linguistic labels. Then, the level of positive (or negative) synergy for every criteria group is calculated using λ-fuzzy measures. The proposed linguistic labels are very simple to use and they provide an adequate abstraction level.

λ-variability provides accurate values of fuzzy measure to express the strength of criteria coalition. This point is relevant because the decision-maker's opinion is properly represented.

By using the Choquet integral it is possible to replace the weighed arithmetic mean to aggregate dependent decision criteria obtaining more accurate results. By comparing the performance of the OWA operator and the Choquet integral (using the proposed method) as aggregation operator, better results can be observed. Clearly, WAM and OWA operators cannot represent this type of situations.

FUTURE WORK

This work is part of a research project related to the study of Decision Support Systems (DSS) and it contains the initial results of the proposed criteria coalition model. Currently, some proposed model improvements are under development (as the use of other mechanisms to obtain fuzzy measures).

In addition, the research team is working on future application of this proposed model to the assessment and evaluation of investment projects for financial organizations.

Furthermore, some interesting topics have been analyzed in order to extend the use of this model with opinion of multiple experts.

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